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LETTER TO THE EDITOR

The critical exponent γ for the three-dimensional Ising model

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Abstract. We have re-examined the high-temperature susceptibility series of the spin-$\frac{1}{2}$ Ising model on the three-dimensional tetrahedron lattice. We conclude that the series data provide strong evidence for the result $\gamma = 1\cdot250$ and are inconsistent with the recent renormalisation group prediction $\gamma = 1\cdot2402 \pm 0\cdot0009$.

There has recently arisen in the literature an apparent conflict regarding the value of the susceptibility exponent γ for the three-dimensional Ising model.

The development of high-temperature series, particularly by the King's College group, has provided a consistent picture of the nature of the critical singularities of the Ising model. In particular the asymptotic form of the susceptibility appears to be a power law

$$\chi \sim C(T-T_c)^{-\gamma}, \qquad T \to T_c+, \tag{1}$$

and series estimates have suggested that $\gamma = 1\cdot250 \pm 0\cdot001$. It has generally been believed that $\gamma = \frac{5}{4}$ exactly.

Recent renormalisation group calculations for the field theory version of the Ising model (Baker *et al* 1978, Le Guillou and Zinn-Justin 1977) have yielded a slightly lower value for γ, namely $\gamma = 1\cdot2402 \pm 0\cdot0009$.

The discrepancy may be due to over-optimistic confidence limits in either or both of the above calculations, or may indicate that the two models are not asymptotically equivalent, as is suggested by more recent work by Baker and Kincaid (1979).

In this Letter we hope to make a modest contribution to this question by presenting some new results from the high-temperature series side. The high-temperature series work has been re-examined in a recent Letter by Gaunt and Sykes (1979), who provide new evidence for the result $\gamma = \frac{5}{4}$ from the series for the simple cubic, face-centred cubic and body-centred cubic lattices. We provide further evidence from the series for a less common three-dimensional lattice, the tetrahedron lattice (also known as the crystobalite or B-site spinel lattice). We have derived and analysed the high-temperature susceptibility series for this lattice in earlier work (Ho-Ting-Hun and Oitmaa 1975, Oitmaa and Ho-Ting-Hun 1976, referred to hereafter as I and II). For reasons which we do not fully understand the series for this lattice seems to be even more regular than those for the more common lattices. In I we made the estimate $\gamma = 1\cdot250 \pm 0\cdot001$, which was biased in the sense that it depended on a prior choice for the critical temperature.

If the asymptotic form (1) is correct then the function $\chi^{1/\gamma}$ will have a simple pole at T_c, which should be revealed by Padé approximants. From the series given in I, expressed in terms of the standard high-temperature variable $v = \tanh(J/kT)$, we have obtained series for $\chi^{1/\gamma}$ for values of γ in the range (1·23, 1·26). For each series we obtain the location of the pole which represents the physical singularity. These are plotted in figure 1 for the highest-order Padé approximants. The 'best' estimate of γ is that for which all Padés give the same estimate of the location of the singularity. As can be seen from the diagram, all of the lines intersect at essentially one point, corresponding to $\gamma = 1{\cdot}250$. It is quite remarkable that for an assumed value of $\gamma = 1{\cdot}250$ all of the central Padés give the same estimate of v_c to within six significant figures, whereas any other choice of γ does not yield this degree of consistency. In addition for γ values outside the range of about (1·24, 1·26) the Padés show spurious singularities, and we regard this as further evidence that the correct γ is very close to, if not identical with, 1·25. Another piece of evidence in favour of $\gamma = 1{\cdot}25$ is the analysis, presented in II, for the next most singular term, where we found consistent evidence that this term diverges with exponent $\gamma_1 = 0{\cdot}25$, thus indicating that the amplitude is analytic at the critical point.

In conclusion we find that analysis of the high-temperature susceptibility series for the $s = \frac{1}{2}$ Ising model on the tetrahedron lattice gives strong support for the hypothesis $\gamma = 1{\cdot}25$, and is inconsistent with the renormalisation group estimate $1{\cdot}2402 \pm 0{\cdot}0009$. Our conclusions thus support those of Gaunt and Sykes (1979), but of course do not resolve the discrepancy between the two approaches.

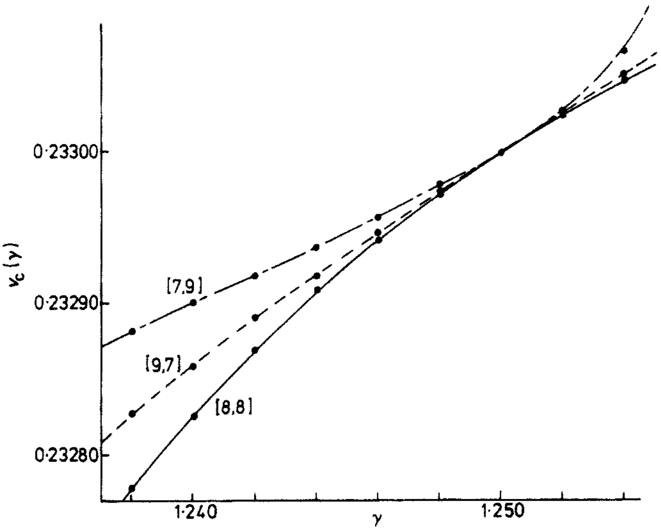

Figure 1. The location of the physical pole $v_c(\gamma)$ from central Padé approximants to $[\chi(v)]^{1/\gamma}$ for different values of γ.

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